Static equilibrium

- Two-dimensional treatment only.
- Conditions for equilibrium of forces acting on a point mass and a rigid body.
- Centre of gravity and its experimental determination. Stability (very briefly).
  - E2. Determination of C.G. of a body of any shape.

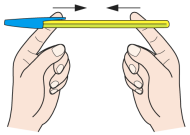
Centre of Gravity

Centre of gravity

The centre of gravity (CG) of an object is the point where the line of action of the weight of the object passes.

move fingers towards each other

centre of gravity = fingers meet (in equilibrium)

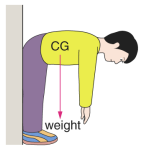


Centre of gravity

Bend over with your legs against wall without toppling?

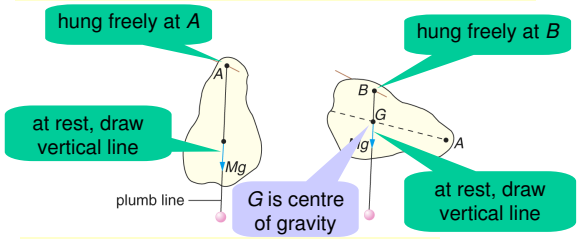
You cannot

Line of action of your weight acts outside the base of your feet and you topple over.

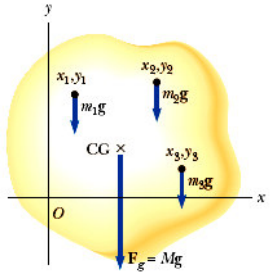


Determine the centre of gravity of a lamina

1. Experimental method



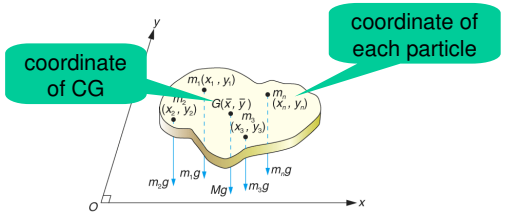
Line of action of its weight must pass through the centre of gravity.



Determine the centre of gravity of a lamina

2. Mathematical method

Assume: Lamina is made up of *n* particles



Determine the centre of gravity of a lamina

2. Mathematical method

Taking moments about the *y*-axis,

$$Mg(\bar{x}) = (m_1g_1)x_1 + (m_2g_2)x_2 + (m_3g_3)x_3 + \ldots + (m_ng_n)x_n$$

$$= \sum_{i=1}^{n}(m_ig_ix_i)$$

$$\bar{x} = \frac{\sum_{i=1}^{n}(m_ig_ix_i)}{Mg}$$

Determine the centre of gravity of a lamina

2. Mathematical method

If g = constant (in a uniform gravitational field), $g_1 = g_2 = g_3 = \ldots = g_n = g$, then

$$\bar{x} = \frac{\sum_{i=1}^{n}(m_ix_i)}{M}$$

Similarly, by taking moments about the *x*-axis,

$$\bar{y} = \frac{\sum_{i=1}^{n}(m_iy_i)}{M}$$

Determine the centre of gravity of a lamina

2. Mathematical method

coordinates of centre of gravity:

$$\bar{x} = \frac{\sum_{i=1}^{n}(m_i x_i)}{M} \qquad \bar{y} = \frac{\sum_{i=1}^{n}(m_i y_i)}{M}$$

Note: These are in fact the coordinates of the centre of mass.

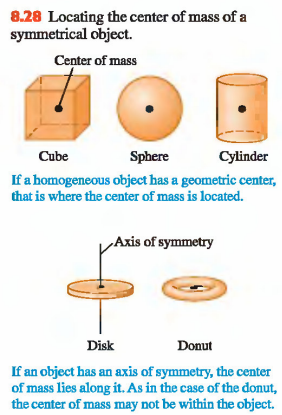
Go to Example 13 Go to Example 14 Go to Example 15

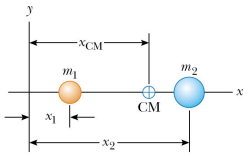
Centre of mass

The centre of mass of an object is the point where the whole mass of the object is assumed to be concentrated.

Centre of mass = Centre of gravity

if the object is small or it is in a uniform gravitational field





The center of mass of two particles of unequal mass on the x axis is located at x_{CM}, a point between the particles, closer to the one having the larger mass.

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

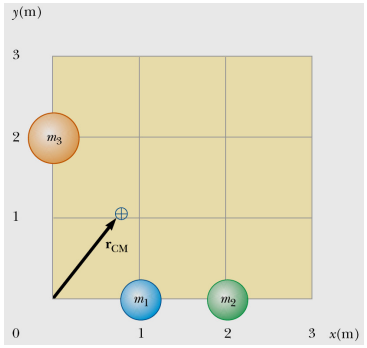
Extending this concept to a system of many particles in three dimensions, the x coordinate of the center of mass of n particles is defined to be

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{m_1 + m_2 + m_3 + \cdots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

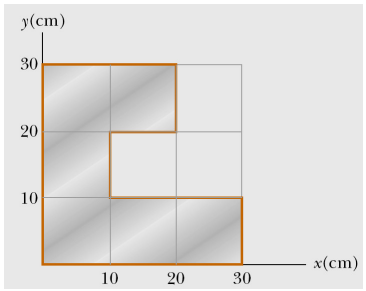
MOTION OF A SYSTEM OF PARTICLES

- ***Total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass.***
- ***The center of mass of a system of particles of combined mass M moves like an equivalent particle of mass M would move under the influence of the resultant external force on the system.***

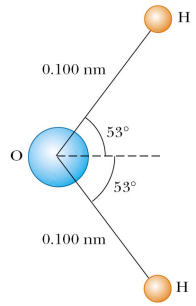
Find the centre of mass of the system of particles shown.



A uniform piece of sheet steel is shaped as shown in Figure. Compute the x and y coordinates of the center of mass of the piece.

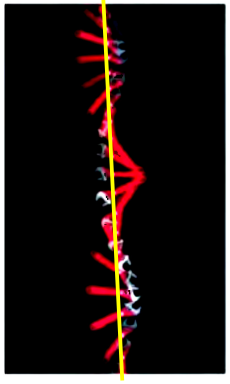


A water molecule consists of an oxygen atom with two hydrogen atoms bound to it. The angle between the two bonds is 106°. If the bonds are 0.100 nm long, where is the center of mass of the molecule?

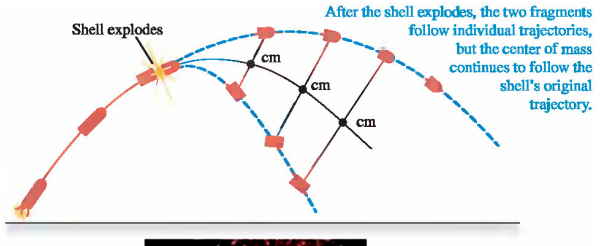


Significance of centre of gravity

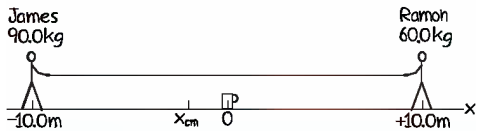
- The total momentum of a system of particles (or a body) is equal to the total mass times the velocity of the center of mass.
- When a body or a system of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.



- The center of mass of this wrench is marked with a white dot.
- The net external rorce acting on the wrench is almost zero.
- As the wrench spins on a smooth horizontal surface, the center of mass moves in a straight line with nearly constant velocity.

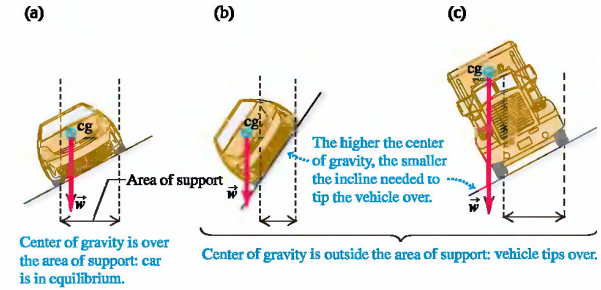


A tug-of-war on the ice



The frozen surface is horizontal and essentially frictionless, so the net external force on the system of James, Ramon, and the rope is zero.
Hence their total momentum is conserved.
Initially there is no motion, so the total momentum is zero; thus the velocity of the center of mass is zero, and the center of mass remains at rest.

Equilibrium and Stability



In (a) the center of gravity is within the area bounded hy the supports, and the car is in equilibrium. The car in (b) and the truck in (c) will tip over because their centers of gravity lie outside the area of support.

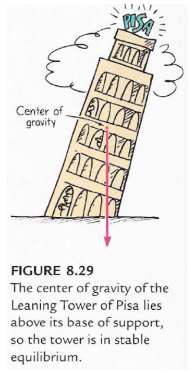


FIGURE 8.29
The center of gravity of the Leaning Tower of Pisa lies above its base of support, so the tower is in stable equilibrium.

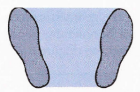
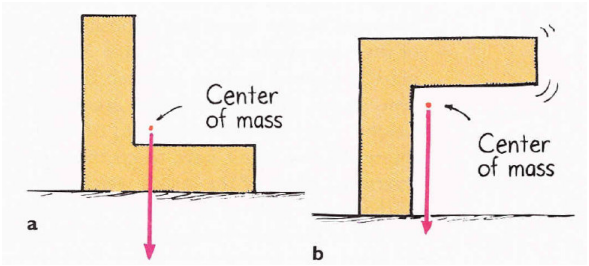
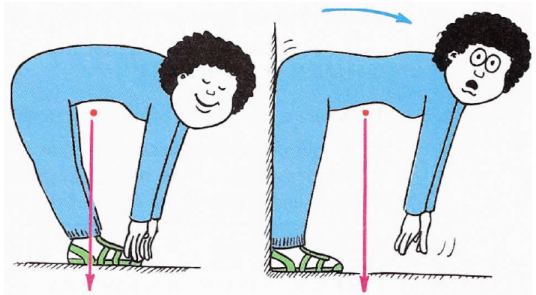
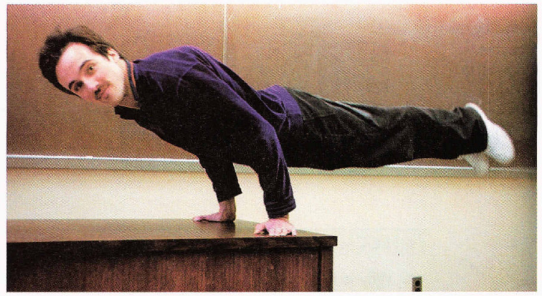


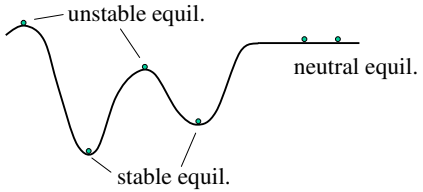
FIGURE 8.30
When you stand, your center of gravity is somewhere above the base area bounded by your feet. Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?





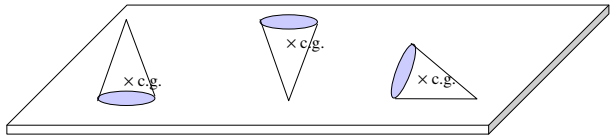


Types of Equilibrium

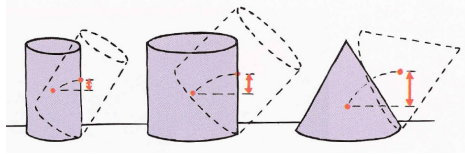


	stable equil.	unstable equil.	neutral equil.
position			
p.e.			

Types of equil. (of a body) in terms of centre of gravity



	stable equil.	unstable equil.	neutral equil.
position of c.g.			
movement of c.g. when rotated about pivot			



Stability is determined by the vertical distance that the center of gravity is raised in tipping. An object with a wide base and a low center of gravity is more stable.

End